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# CORE–ANNULAR OIL/WATER FLOW: THE TURBULENT-LUBRICATING-FILM MODEL AND MEASUREMENTS IN A 5 cm PIPE LOOP

R. V. A. OLIEMANS, G. OOMS, H. L. WU and A. DUIJVESTIJN
Koninklijke/Shell-Laboratorium, Shell Research B.V., Amsterdam, The Netherlands

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**Abstract**—Core-flow tests with a 3000 mPa s fuel oil in a 5 cm test facility have revealed important information on the amplitudes and lengths of waves at the oil/water interface. The wavelengths vary considerably with water fraction and oil velocity. Moreover, the flow in the water annulus is turbulent. A previously developed theoretical model for steady core–annular flow in pipes has been extended by incorporating the effect of turbulence in the water film surrounding the oil core. The adapted model predicts the pressure-gradient increase with oil velocity correctly, provided that actual wave amplitudes and wavelengths observed during these tests are used as input data. The possible contribution of inertial effects is discussed.

## INTRODUCTION

The technique based on core–annular flow is attractive for transporting viscous oils in pipelines (viscosities in excess of 1000 mPa s), not least because the pumping power required is comparable to that for pure water flow. The principle is that a thin water film is introduced between the oil and the pipe wall. This film acts as a lubricant, giving a pressure-gradient reduction. Measurements in 2.5, 5 and 20 cm dia pipes in the laboratory have shown that, under certain conditions, it is possible to use very thin water films (Oliemans & Ooms 1986). For crude oils with viscosities exceeding 2000 mPa s, stable operation has proved feasible with as little as 2% water.

Figures 1a–c, photographs of core flow through a 5 cm transparent pipe, illustrate clearly the features of this flow mode. It can be seen that the oil core is kept away from the pipe wall. The oil/water interface typically forms a wavy pattern. With the aid of a low-cost additive in the film water, which renders the pipe wall oil-repellant, it is possible to stop the core flow and restart after days of standstill without fouling problems.

One of the central questions regarding core–annular flow in a horizontal pipe is how the buoyancy force on the core, present due to the difference in density between oil and water, is counterbalanced. The lubricating film model introduced by Ooms *et al.* (1984) provides a possible answer to this question. In this model the oil viscosity is assumed to be so high that any flow in the core, and hence any variation in the oil/water interface form with time, may be neglected. Thus the core is assumed to be solid and the interface to be a solid/liquid interface. When used to compute pressure gradients for stable core-annular flow in pipelines the model requires oil/water interfacial wave data as input for its calculations. It has been demonstrated that pressure gradients calculated according to this model are quite sensitive to changes in the amplitude, length and asymmetry of waves that are invariably observed at the oil/water interface during operation in the core-flow mode. From previous laboratory experiments no data on wave amplitudes could be derived, while the wavelength could be estimated at only one velocity condition.

In the present paper core-flow tests with a 3000 mPa s fuel oil in a 5 cm test facility are presented. The experiments supply important information on the amplitudes and lengths of waves at the oil/water interface. The pressure-gradient data obtained suggest that the effect of turbulence in the water film is most likely not restricted to the lower part of the pipe, so the lubricating-film model has been adapted accordingly. The set of model equations, extended to include the effect of

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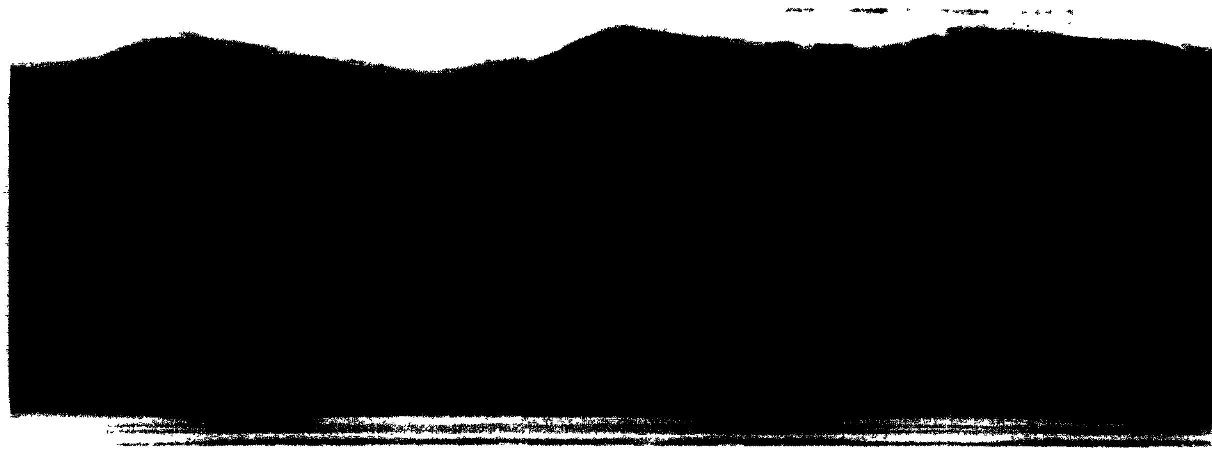
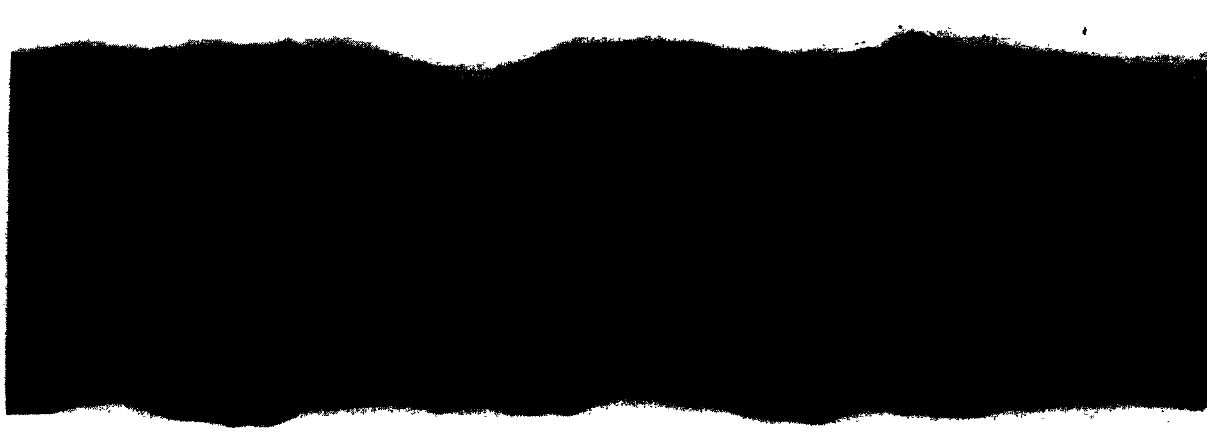


(a) $V_{SO} = 0.5$ m/s, $C_W = 0.20$.

(b) $V_{SO} = 1.0$ m/s, $C_W = 0.03$.



(c) $V_{SO} = 1.0$ m/s, $C_W = 0.20$.

Direction of flow ←

Figures 1a–c. Core flow of a 3000 mPa s fuel oil in the 5 cm test loop.

## EXPERIMENTAL INVESTIGATION

A 5 cm horizontal pipe loop with a total length of 16 m was used for core-flow measurements. Not only were pressure gradients measured, as in the past; the shape, amplitude and length of the waves also received attention, together with film thickness variation around the pipe circumference and water hold-up. The interfacial wave and water annulus data, recorded with the aid of specially developed instrumentation, are currently being analysed and will be the subject of a future paper. Here, for a range of oil velocities and water fractions, pressure-loss data and some interesting information derived from photographs taken during the core-flow tests (figures 1a–c) will be presented.

Photographs were taken during core-flow operations with a 3000 mPa s fuel oil, which has a density 20 kg/m$^3$ less than that of water. For this purpose a transparent section was installed in the pipe loop, consisting of a Perspex tube immersed in a plane rectangular trough of the same material filled with glycerine. The combination of the liquid and the Perspex precluded the problem of optical distortion due to diffraction. With a horizontally aligned camera, the upper and lower wave profiles of the core could be photographed. Figures 1a–c show the oil flow from right to left, with superficial velocities ($V_{SO}$) and input water fractions ($C_W$) of: (a) 0.5 m/s and 0.20; (b) 1 m/s and 0.03; and (c) 1 m/s and 0.20.

When the photographed profiles were copied and analysed, the following points were noted:

—The autocorrelation functions of most of the profiles were weak, and the profiles were not regular; in each case, though, a dominant wavelength could be found.
—There was no interrelation between an upper profile and its corresponding lower profile because the two could not be cross-correlated.
—For input water fractions $<0.15$, the difference in water-layer thickness between the upper and lower sides was $<5\%$ of the pipe diameter.

Figure 2, a plot of the wavelengths determined from the photographic material, shows a decrease in wavelength with increasing superficial oil velocity and/or decreasing water fractions. Wavelengths were seen to vary from 6 to 60 mm, while wave amplitudes were of the order of 1–2 mm. It is expected that the wave ripple formation can be related to the distance from the pipe wall. As shown in figure 3, wavelengths ($l$) determined for various water fractions can be plotted on a single line as a function of superficial oil velocity by using the ratio $l/[R(1-\sqrt{1-C_W})]$, where $R$ is the pipe radius [$R(1-\sqrt{1-C_W})$ is in fact the average water film thickness if no relative velocity

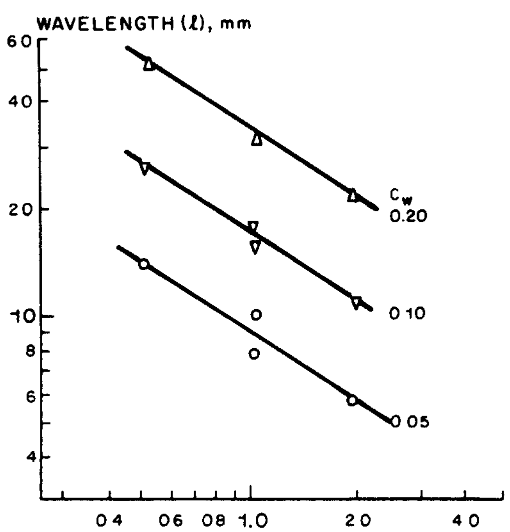



Figure 2. Plot of wavelength ($l$) for different input water fractions ($C_W$) measured in the 5 cm test loop.

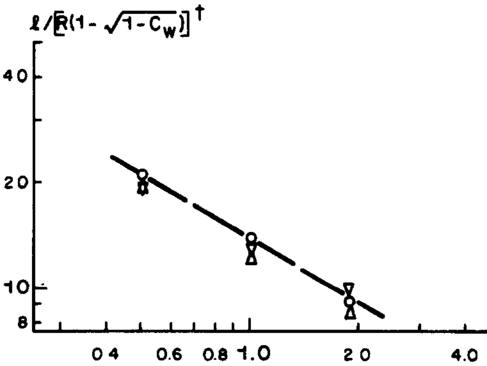



Fig. 3. Correlation of $l$ and $C_W$ with superficial oil velocity ($V_{SO}$). †Wavelength normalized against annular thickness.

existed between the two fluids, oil and water]. The correlation for the wavelength shown in figure 3 can be expressed as

$$l = 13.6 \cdot R(1 - \sqrt{1 - C_W}) \cdot V_{SO}^{-0.594}, \qquad [1]$$

where

$l$ = wavelength (mm),
$R$ = pipe radius (mm),
$C_W$ = input water fraction

and

$V_{SO}$ = superficial oil velocity (m/s).

Water hold-ups ($H_W$) could only be determined from the photographs. The values obtained correlate with the water fraction $C_W$ and lead to the following empirical expression:

$$H_W = C_W[1 + (1 - C_W)^5]. \qquad [2]$$

Core-flow pressure gradients measured for water fractions ranging from $C_W = 0.05$ to $C_W = 0.20$ are plotted against mixture velocity in figure 4. For comparison, a curve for pressure gradients calculated for pure water flow is also given. At velocities >0.5 m/s the core-flow pressure gradient varies with the mixture velocity to the power 1.8, i.e. typical for turbulent flow. There is only a slight dependence on water fraction.

## THEORETICAL MODEL

The experimental observation that turbulence in the water film is of importance forced us to reconsider the theoretical model for steady core flow in horizontal pipes. A study was initiated of the possibility of extending the core-flow model, which is based on the hydrodynamic lubrication theory (Ooms et al. 1984), to a model in which turbulence in the water film is fully taken into account. The approach considered here is one of generalizing the flow equations for the water film to those for the turbulent lubrication theory.

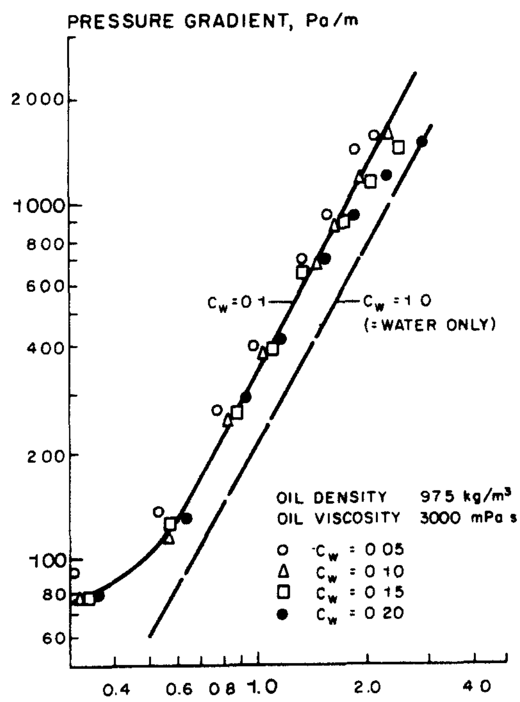



Figure 4. Pressure gradient measured in the 5 cm test loop for various water fractions and oil velocities.

The starting-point for the turbulent-lubricating-film model for steady core flow in a pipe is the following set of equations derived from the continuity equation and the time-independent equations of motion (in cylindrical coordinates $x$, $y$ and $\theta$ for the axial, radial and azimuthal directions, respectively):

$$-\frac{\partial}{\partial y}(uR)+\frac{\partial v}{\partial \theta}+R\frac{\partial w}{\partial x}=0, \tag{3}$$

$$\frac{\partial \phi}{\partial y}=0 \tag{4}$$

$$\frac{\partial}{\partial y}\left(\mu^*\frac{\partial v}{\partial y}\right)=\frac{1}{R}\frac{\partial \phi}{\partial \theta} \tag{5}$$

and

$$\frac{\partial}{\partial y}\left(\mu^*\frac{\partial w}{\partial y}\right)=\frac{\partial \phi}{\partial x}, \tag{6}$$

where

$u, v, w$ = the velocity components of water in the $y$, $\theta$ and $x$ directions, respectively (m/s),
$\phi = p + \rho g r \cos\theta + 2/3\,\rho k$, pressure variable (Pa),
$\rho$ = density (kg/m$^3$),
$k$ = turbulent kinetic energy per unit mass (J/kg),
$r = R - y$, radial length (m)

and

$g$ = acceleration due to gravity (m/s$^2$).

In [5] and [6] the viscosity $\mu^*$ consists of a turbulent and a molecular part: $\mu^* = \mu_t + \mu_m$, where $\mu_t$ is a function of $x$, $y$ and $\theta$. The above equations for $u$, $v$, $w$ and $\phi$ have been derived from the continuity equation and the Navier–Stokes equations on the same assumption as in our previous work (Ooms *et al.* 1984), i.e. that not only is the thickness $h$ of the water film much smaller than both the pipe radius $R$ and the length $l$ of the wave in the oil core, but also the group $(\rho Wh/\mu^*)\cdot(h/l) < 1$ (here $W$ is the velocity of the oil core). As before, the assumption is that the oil core is rigid (with a "frozen" ripple at its surface).

The basic equation for $\phi$, a function of $x$ and $\theta$ only (according to [4]), is known as the Reynolds equation in hydrodynamic lubrication theory and is derived by substituting, in [3], expressions for the velocities $v$ and $w$ obtained from integration of [5] and [6]. For computational convenience, the viscosity $\mu^*$ has been replaced by its average ($\bar{\mu}$) over the radial distance, $y$:

$$\bar{\mu}=\frac{\int_0^h \mu^*\,\mathrm{d}y}{h}. \tag{7}$$

Thus, the viscosity $\bar{\mu}$, like $\phi$, is now a function of $x$ and $\theta$ only. The expression for the turbulent Reynolds equation—the partial differential equation describing the pressure generated in a turbulent thin-film flow—then becomes

$$\frac{\partial}{\partial \theta}\left(\frac{h^3}{\bar{\mu}R^2}\frac{\partial \phi}{\partial \theta}\right)+\frac{\partial}{\partial x}\left(\frac{h^3}{\bar{\mu}}\frac{\partial \phi}{\partial x}\right)=6W\frac{\partial h}{\partial x}. \tag{8}$$

This is a generalization of the equation used in the earlier version of the core-flow model (Ooms *et al.* 1984, [8]).

For the description of turbulence via the viscosity $\mu_t$ in $\mu^*$ a simple turbulence model, Prandtl's mixing-length model, has been chosen:

$$\nu_t=\frac{\mu_t}{\rho}=l_m^2\left|\frac{\mathrm{d}w}{\mathrm{d}y}\right|. \tag{9}$$

Here the expression for the mixing length $l_m$ is that given by Nikuradse for channel flow with Van Driest's hypothesis for the effect of the wall:

$$l_m = h\left[0.14 - 0.08\left(1-\frac{y}{h}\right)^2 - 0.06\left(1-\frac{y}{h}\right)^4\right]\left[1-\exp\left(\frac{-y^+}{A}\right)\right], \tag{10}$$

where

$$y^+ = \frac{y\,\mathrm{Re}^*}{h} \tag{11}$$

with

$$\mathrm{Re}^* = 0.11\,(\mathrm{Re})^{0.911},$$

$$\mathrm{Re} = \frac{Wh}{\nu_m},$$

$W$ = maximum water velocity ( = oil core velocity)

and

$A = 26$ (Van Driest constant).

The velocity gradient $\mathrm{d}w/\mathrm{d}y$ in [9] can be computed according to

$$\frac{\mathrm{d}w}{\mathrm{d}y} = \frac{w^*}{\kappa y}$$

with $\kappa$ (Von Kármán constant) $= 0.35$ and $w^* = \nu_m \mathrm{Re}^*/h$.

The next two steps in the turbulent-lubricating-film model for steady core flow are the determination of the position of the oil core (its eccentricity) and the corresponding pressure gradient. The equation stating that in steady core–annular flow the buoyancy force on the core is counterbalanced by hydrodynamic lubrication forces (with due account taken of turbulence in the water annulus) reads as follows:

$$2R\int_0^\pi\int_0^l \phi\cos\theta\,\mathrm{d}\theta\,\mathrm{d}x - \int_0^\pi\int_0^l 2\alpha\frac{\partial\phi}{\partial\theta}\sin\theta\,\mathrm{d}\theta\,\mathrm{d}x = (\rho-\rho_C)g\pi l R^2 \tag{12}$$

where

$$\rho_C = \text{core density}$$

and

$$\alpha = \frac{\displaystyle\int_0^h \frac{y}{\mu^*}\,\mathrm{d}y}{\displaystyle\int_0^h \frac{1}{\mu^*}\,\mathrm{d}y}.$$

This equation differs from the one used in the earlier version of the lubricating-film model (Ooms *et al.* 1984, [22]), in that the film thickness $h$ in the integrand of the second integral on the l.h.s. is replaced by $2\alpha$, a function of the turbulent viscosity (note that when $\mu_t = 0$, i.e. $\mu^* = \mu_m = \text{const}$, $2\alpha = h$). The equation for the pressure gradient in steady core flow is affected in a similar manner. It differs from the expression used earlier in that $h$ and $\mu_m/h$ are replaced by the turbulent-viscosity-dependent functions $2\alpha$ and $1/\beta$, respectively:

$$R\int_0^\pi\int_0^l 2\alpha\frac{\partial\phi}{\partial x}\,\mathrm{d}\theta\,\mathrm{d}x + 2RW\int_0^\pi\int_0^l \frac{1}{\beta}\,\mathrm{d}\theta\,\mathrm{d}x = \pi R^2(\phi_2-\phi_1), \tag{13}$$

where

$$\beta = \int_0^h \frac{1}{\mu^*}\,\mathrm{d}y.$$

The way in which turbulence effects are introduced into the lubricating-film model for core–annular flow is a very simple one, not very different from the mixing-length approach to turbulent lubrication proposed by Constantinescu (1959). However, it may well be sufficient for our needs, since Launder & Leschziner (1978) have shown that, for finite-width thrust bearings, the much more sophisticated $k$–$\epsilon$ turbulence model leads to effects of turbulence in lubricating films similar to those predicted by Constantinescu.

An aspect that warrants critical examination is the validity of the assumption that the group $(\rho Wh/\mu^*)\cdot(h/l)$ is $<1$, for the purposes of which inertial effects are disregarded. Should this assumption prove invalid, allowance for fluid inertia will have to be incorporated in the turbulent-lubricating-film model. This problem has already been addressed by King & Taylor (1977) and Launder & Leschziner (1978) for a plane inclined slider thrust bearing operating with a turbulent film. They conclude that the effect of fluid inertia relative to non-inertial flow conditions is an increase in load capacity and a relatively small increase in frictional traction on the moving surface. Lubrication with comparable viscous and inertia forces has been studied by Tuck & Bentwich (1983).

Only comparison with measured core-flow data, however, will reveal whether or not refinements (a more sophisticated turbulence model or allowance for fluid inertia) have to be incorporated in the turbulent-lubricating-film model for core-flow applications.

## COMPARISON OF EXPERIMENTAL RESULTS WITH MODEL PREDICTIONS

For steady core flow of a 3000 mPa s oil in the 5 cm test facility with an input water fraction of 0.10 calculations have been performed with two versions of the lubricating-film model, one without turbulence in the water film and one including turbulence effects, as described above. In these calculations the original default values were used for wave amplitude (0.075 mm) and wavelength (20 mm). The results of the calculations are shown in figure 5 as --- and -·- lines, respectively. At a superficial oil velocity of nearly 2 m/s, the effect of turbulence can lead to a pressure-gradient increase of some 30%. This value is in line with results obtained with an intermediate model with turbulence effects restricted to the lower part of the water annulus; at this particular wave amplitude/wavlength combination the position of the oil core in the pipe is highly eccentric, resulting in a very thin water film with hardly any turbulence in the upper part of the annulus.

Compared to measured pressure gradients, also shown in figure 5, neither calculation gives a fair prediction. Subsequently, we performed calculations with the two versions of the lubricating-film

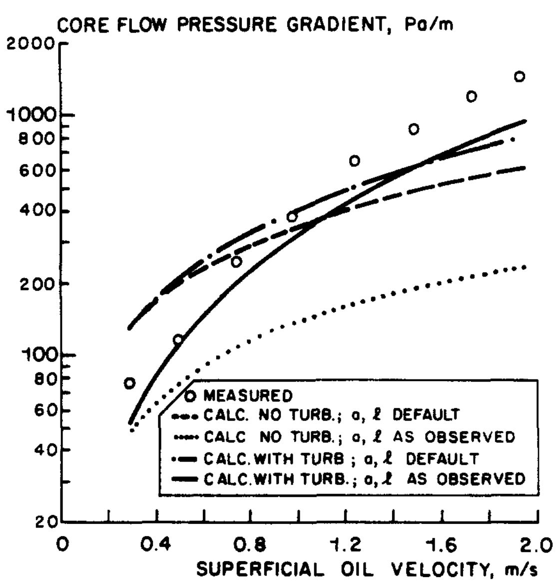



Figure 5. Measured and calculated core-flow pressure gradients for a 3000 mPa s oil with a water fraction of 0.10 in the 5 cm test loop. ($a$ = wave amplitude, $l$ = wavelength).

model, applying wave data that had been observed during the tests. At a water fraction of 0.10, the observed wave amplitude was 2 mm, while the wavelengths varied from 35 mm at a superficial oil velocity of 0.3 m/s to 11 mm at 1.95 m/s. For these wave amplitude/wavelength combinations the position of the oil core in the pipe is much less eccentric than in the previous case. As a consequence, the effect of turbulence on pressure gradients is much stronger, since it plays a role in a larger part of the water annulus. The results of the calculations are shown in figure 5 as —— (with turbulence) and . . . (without turbulence) lines. As illustrated by the —— curve in figure 5, the use of the observed wave data in the turbulent-lubricating-film model brings the pressure-gradient predictions much more closely into line with the measurements. The results of calculations with the original model (i.e. without turbulence) when the observed wave data for amplitude and length are used give pressure gradients that are very much lower.

The promising results with the new model for core flow reveal the importance of turbulence in the water film and of a proper description of the waves on the oil core. Still, as shown in figure 5, even with the turbulent-lubricating-film model pressure gradients are systematically under-predicted, although the dependence on oil velocity is correctly represented. This underprediction could be caused by the fact that the model neglects the contribution of inertial effects. From Launder & Leschziner (1978) we may infer that an increase of some 30% in the pressure gradient, due to inertial effects, is not unrealistic. For a better distinction between the effects of fluid inertia and interfacial waves on calculated pressure gradients, interfacial wave data more accurate than those derived from photographs are needed. The collection and analysis of dedicated measurements of film-thickness variation around the pipe circumference currently in progress will improve the quality of the wave data. Meanwhile, the sensitivity of pressure-gradient predictions to changes in wave data will have to be reassessed, now that it is known that turbulence plays an important role. In particular, the wave shape (asymmetry, for which a value of 0.8 was used here, corresponding to a wave with a long gradual front and a steep tail) will have to be reconsidered.

For application of the model to larger sizes of pipe (dia > 5 cm) the proper scaling rules for the wave geometry must be determined. Data on waves in larger pipes are currently being collected for this purpose.

## CONCLUSIONS

The application of the turbulent-lubricating-film model and the 5 cm core-flow test results lead to the following conclusions:

(i) The core-flow tests with a 3000 mPa s fuel oil in a 5 cm pipe showed that the amplitudes and lengths of waves on the oil/water interface vary with water fraction and oil velocity.

(ii) For a correct prediction of the pressure-gradient increase with oil velocity, using the observed wave data, the lubricating-film model had to be extended to include the effect of turbulence in the water film.

(iii) The effects of inertia may play a role in removing the systematic underprediction of the pressure gradient. More accurate film-thickness data, currently being measured, are required to establish whether this is indeed the case.

(iv) In view of the influence of wave data on model predictions, the application of the model to larger pipes requires accurate scaling rules for wave geometry. These have to be determined from dedicated experiments.

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